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Research Article

Odd and Even Codes Existence with Repetitive Formation Codes On the Ring $\mathbf{F_2[v]/\langle v^2 - 1\rangle}$

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Abstract – In this study, on the ring $F_2[v]/\langle v^2 - k\rangle$ to be in the situation condition with $k = 1$ has been studied . This the four elements ring special conditions under code clusters written (special generators by giving). More later this four elemental set with binary fields between relationship was established . More before published by M. Özkan and F. Öke titled Repeat Codes, Even Codes, Odd Codes and Their Equivalence [16] from his work was used. In [16], the study $k = 0$ aspect has been studied . Here we wrote working different of type ring structure with different generators with codes with $k = 1$ condition . After that this of codes, odd and even whether they are double or not is classified . Defined generators with get made codes; odd code or which odd to the code equivalent is similar way even code or which even to the code equivalent is presented. Repetitive formation of codes, odd and even with codes similarities and from their equivalences has been mentioned. More later examples by giving studied code of sets Which of type to codes opposite was brought were studied. Examples with repetitive formation of codes and equivalent of codes application were given.

Key Words – Repeated Formation Codes , Odd Codes , Even Codes , Hadamard Codes , Generator Matrices .

I. INTRODUCTION

Published By M. Özkan and F. oke titled Repeat Codes, Even Codes, Odd Codes and Their Equivalence [16] from his work was used . In [16], the study $k = 0$ it was studied as. Here, we worked with $k = 1$ status. In literature, 4 elements chain rings different one a lot coding theory in his article structure aspect has been studied . For example by Zhu, S., Wang, Y. and M. Sji, at [11], again by A. Bonnecaze and P. Udaya at [4], By M. Özkan at [16] and [14] in his works .

$F_2[v]/\langle v^2 - k\rangle$ in the ring $k = 1$, the ring has been studied for the situation on R in case of $v^2 = 1$. Moreover on the ring defined cyclic codes of codes and quasi-cyclic codes with relationship has been given. Moreover This ring on odd and even of codes definitions has been made .

From here ,

k is equal to one while $v^2 = 1$. Well;

$$F_2[v]/\langle v^2 - k\rangle = F_2[v]/\langle v^2 - 1\rangle$$

$= \{a_0 + a_1 v + \langle v^2 - 1\rangle | a_0, a_1 \in F_2\}$ is possible.

This ring elements;

$$a_0 = 0, a_1 = 0 \; için \; 0 + 0v = 0$$
$$a_0 = 1, a_1 = 0 \; için \; 1 + 0v = 1$$
$$a_0 = 0, a_1 = 1 \text{ için } 0 + 1v = v$$
$$a_0 = 1, a_1 = 1 \text{ için } 1 + 1v = 1 + v$$

form is found. The ring $F_2[v]/\langle v^2 - 1\rangle$ of elements $\{0,1,v,1+v\}$ consists of a set. This cluster $F_2 + vF_2 = \{0,1,v,1+v\}$ set below defined $+$ and $\cdot$ is a ring with its operations.

$+$	0	1	v	$1+v$
0	0	1	v	$1+v$
1	1	0	$1+v$	v
v	v	$1+v$	0	1
$1+v$	$1+v$	v	1	0

$\cdot$	0	1	v	$1+v$
0	0	0	0	0
1	0	1	v	$1+v$
v	0	v	1	$1+v$
$1+v$	0	$1+v$	$1+v$	0

$F_2[v]/\langle v^2 - 1\rangle$ ring is isomorphic to the ring $F_2 + vF_2$. In case $v^2 = 1$, $R = F_2 + vF_2$ is the ring with ideals of $\langle 0\rangle = \{0\}, \langle 1+v\rangle = \{0, 1+v\}$ and $\langle v\rangle = \langle 1\rangle = R$. Ideals of this ring have between

$\langle 0\rangle \subseteq \langle 1+v\rangle \subseteq \langle v\rangle = \langle 1\rangle = R$

relationship and R is a local ring.

II. PRELIMINARIES

In this section, the definitions of the weight function and minimum distance function on the ring are given. In addition, the definition of Gray transform, which establishes the relationship between the ring and the object is presented.

2.1 Definition: $R = F_2 + vF_2$ on the ring;

for every $r \in R$

$$w_{L_R}(x) = \begin{cases} 0 \; , & x = 0 \\ 1 \; , & x = 1, v \\ 2 \; , & x = 1 + v \end{cases}$$

shaped defined to function is called the Lee weight function on R.

In this case, equality is ensured for $w_{L_R}(r) = \sum_{i=1}^{n} w_{L_R}(r_i)$ all $r = (r_1, r_2, \dots, r_n) \in R^n$,

$$F_2[v]/\langle v^2 - k\rangle = F_2[v]/\langle v^2 - 1\rangle$$
$$= \{a_0 + a_1 v + \langle v^2 - 1\rangle | a_0, a_1 \in F_2\}$$

of the ring elements;

$$a_0 = 0, a_1 = 0 \; için \; 0 + 0v = 0$$
$$a_0 = 1, a_1 = 0 \; için \; 1 + 0v = 1$$
$$a_0 = 0, a_1 = 1 \text{ için } 0 + 1v = v$$

$a_0 = 1, a_1 = 1$ için $1 + 1v = 1 + v$ form is found.

2.2 Definition: On F_2 the object;

The function defined in the form F_2 for $c \in F_2$, $w_H(0) = 0$, $w_H(1) = 1$ each is called the Hamming weight function. This case $w_H(r) = \sum_{i=1}^{n} w_H(c_i)$ for every $c = (c_1, c_2, \dots, c_n) \in {F_2}^n$ is possible.

2.3 Definition: On R^n, distance between two vectors $a, b \in R^n$ and $a \neq b$
distance between vectors $d_{L_R}(a,b) = w_{L_R}(a-b)$ to be about, a C Minimum Lee distance of code
$d_{L_R}(C) = min\{d_{L_R}(a,b) |\ a \neq b\ ve\ \ a,b \in C\}$
form is defined .

${F_2}^n$ a similar definition is given for Hamming weight. On ${F_2}^n$, one it is indicated by C the minimum Hamming distance of the code $d_H(C)$.

2.4 Definition: In case $v^2 = 1$, let ring is $R = F_2 + vF_2$;

$$\Phi: R^n \to {F_2}^{2n}$$

$$(r_1, r_2, \dots, r_n) \to \Phi(r_1, r_2, \dots, r_n)$$

be given. From here;

$$\Phi(r_1, r_2, \dots, r_n) = \Phi(a_1 + vb_1, a_2 + vb_2, \dots, a_n + vb_n) = (a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n)$$

is possible . Like this $1 \leq i \leq n$ ve $a_i, b_i \in F_2$ for $r_i = a_i + vb_i \in R$. Form defined Φ to the anniversary R. It is called Gray transformation on the ring. This definition where $v^2 = 1$, Gray transformation of a code of length n, on the ring in the case $F_2 + vF_2$ under Φ image length $2n$ is a binary code . On R^n defined d_{L_R} Lee distance and on ${F_2}^{2n}$defined d_H Hamming distance, there is a relationship$d_{L_R}(a,b) = d_H(\Phi(a), \Phi(b))$ between each $a, b \in R^n$. From here Φ can be seen that Gray transforms are isometrics.

III. HADAMARD CODES AND REPETITIVE FORMATION CODES

3.1 Definition : On the ring $R = F_2 + vF_2$ in the case $v^2 = 1$; for $\alpha_1, \alpha_2 \in Z_+ \cup \{0\}$, first line components $\{1\}$ other row elements from the set, $\alpha_2 = 0$ while $\{0, 1, v, 1+v\}$ from the set and $\alpha_1 = 0$ while $\{0, 1+v\}$, the columns are also sorted lexicographically by being selected from the set to the relation according to sorted in the form and line number $\alpha_1 + \alpha_2 + 1$ the which one. special aspect created ${N_R}^{\alpha_1,\alpha_2}$ to the matrix R generator matrix is called .

Below ${N_R}^{\alpha_1,\alpha_2}$ matrices for a few example has been given .

$${N_R}^{0,0} = [1], \qquad {N_R}^{0,1} = \begin{bmatrix} 1 & 1 \\ 0 & 1+v \end{bmatrix},$$

$${N_R}^{0,2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1+v & 1+v \\ 0 & 1+v & 0 & 1+v \end{bmatrix},$$

$${N_R}^{0,3} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1+v & 1+v & 1+v & 1+v \\ 0 & 0 & 1+v & 1+v & 0 & 0 & 1+v & 1+v \\ 0 & 1+v & 0 & 1+v & 0 & 1+v & 0 & 1+v \end{bmatrix},$$

$${N_R}^{1,0} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & v & 1+v \end{bmatrix},$$

$${N_R}^{2,0}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & v & v & v & v & 1+v & 1+v & 1+v & 1+v \\ 0 & 1 & v & 1+v & 0 & 1 & v & 1+v & 0 & 1 & v & 1+v & 0 & 1 & v & 1+v \end{bmatrix},$$

$${N_R}^{1,1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & v & v & 1+v & 1+v \\ 0 & 1+v & 0 & 1+v & 0 & 1+v & 0 & 1+v \end{bmatrix}.$$

3.2 Definition : $R = F_2 + vF_2$ the ring in the case of $v^2 = 1$, for each $\alpha_1, \alpha_2 \geq 0$ integer produced by ${C_R}^{\alpha_1,\alpha_2} = \{(c_1, c_2).{N_R}^{\alpha_1,\alpha_2} |\ c_1 \in R^{\alpha_1+1}, c_2 \in {F_2}^{\alpha_2}\}$ the code ${N_R}^{\alpha_1,\alpha_2}$generator matrix written in the form, the length $n = 2^{2\alpha_1 + \alpha_2}$ is $(n, 4n, n)$ parameterized code is called.

3.3 Theorem [16]: Φbe the Gray transformation on R^n. ${C_R}^{\alpha_1,\alpha_2}$, ${N_R}^{\alpha_1,\alpha_2}$ generator matrices written code to be about $\Phi({C_R}^{\alpha_1,\alpha_2})$. The code is a Hadamard code $F_2$with $(2n, 4n, n)$ parameters.

3.4 Corollary [16]: $\alpha_1 = \alpha_2 = 0$ status outside ; It becomes $({C_R}^{\alpha_1,\alpha_2})^{\perp}$ a parameter of the dual code $\left(n, \frac{4^n}{4n}, 4\right)$. Additionally, $\Phi(({C_R}^{\alpha_1,\alpha_2})^{\perp})$ the code $\left(2n, \frac{4^n}{4n}, 4\right)$ becomes a parameterized code.

3.5 Definition: Let $n = 2^{2\alpha_1,+\alpha_2}$, on the ring $R = F_2 + vF_2$ $(v^2 = 1)$, ${C_R}^{\alpha_1,\alpha_2}$ be a linear code of length $(v^2 = 1)$.

$$\tau_1 : R^n \to R^n$$
$$(c_1, c_2, \dots, c_n) \to \tau_1(c_1, c_2, \dots, c_n)$$

be given. From here $\tau_1(c_1, c_2, \dots, c_n) = (c_n, c_1, \dots, c_{n-1})$ is possible.

This permutation for $\tau_1({C_R}^{\alpha_1,\alpha_2}) = {C_R}^{\alpha_1,\alpha_2}$, if the condition is provided, ${C_R}^{\alpha_1,\alpha_2}$ is called a cyclic code on R.

3.6 Definition : Let $n = 2^{2\alpha_1,+\alpha_2}$, there D^{α_1,α_2} be a linear code of length $2n$ on F_2.

$$\sigma^{\otimes 2} : {F_2}^{2n} \to {F_2}^{2n}$$
$$(d_1, d_2, \dots, d_{2n}) \to \sigma^{\otimes 2}(d_1, d_2, \dots, d_{2n})$$

be given. Like this $\sigma^{\otimes 2}(d_1, d_2, \dots, d_{2n})$

$$= (d_n, d_1, \dots, d_{n-1}, d_{2n}, d_{n+1}, \dots, d_{2n-1})$$

is possible. This permutation for $\sigma^{\otimes 2}(D^{\alpha_1,\alpha_2}) = D^{\alpha_1,\alpha_2}$, if the condition is provided, D^{α_1,α_2} is called order of two quasi cyclic code on ${F_2}^{2n}$.

3.7 Proposition: τ_1 a permutation on R^n, $\sigma^{\otimes 2}$ a permutation transform on ${F_2}^{2n}$, the Gray Φ transform from R^n to ${F_2}^{2n}$ defined above $\sigma^{\otimes 2} \circ \Phi = \Phi \circ \tau_1$ equality is provided.

3.8 Theorem [14] : In case $\alpha_2 \neq 0$, Hadamard code obtained with the generator matrix is ${N_R}^{\alpha_1,\alpha_2}$ a quasi-cyclic code of order of two .

3.9 Example : ${C_R}^{0,1}$ code to determine for ${N_R}^{0,1}$ generator matrix Let's write. With ${N_R}^{0,1} = \begin{bmatrix} 1 & 1 \\ 0 & 1+v \end{bmatrix}$,

${C_R}^{0,1} = \{(c_1, c_2). {N_R}^{0,1} | \ c_1 \in R, \ c_2 \in F_2\}$ is in the form.

From here ;

$${C_R}^{0,1} = (c_1, c_2). \begin{bmatrix} 1 & 1 \\ 0 & 1+v \end{bmatrix} \Rightarrow c_1, c_1 + c_2.v \mid c_1 \in R = \{0,1,v,1+v\}, \ c_2 \in F_2 = \{0,1\}$$

${C_R}^{0,1} = \{00, 01+v, 11, 1v, vv, v1, 1+v1+v, 1+v0\} \subseteq R^2$ of the code elements for $d_{L_R}({C_R}^{0,1}) = n = 2$ and $|{C_R}^{0,1}| = 4n = 4.2 = 8$ happens.

${C_R}^{0,1}$ of the code parameter $(n, 4n, n) = (2,8,2)$ is in the form. ${C_R}^{0,1}$ Gray image of the code if taken ; From here $\Phi({C_R}^{0,1}) = \{0000,0011,1111,1100,0101,0110,1010,1001\} \subseteq F_2^4$ is the code .

$\Phi({C_R}^{0,1})$ of the code parameter $(2n, 4n, n) = (4,8,2)$ parameterized one hadamard is the code .

Other from the side $M = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, the matrix is obtained by writing $M_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $-1 \to 1$ in the Hadamard matrix $1 \to 0$, which is a normalized Hadamard code. M vectors and 10 are obtained from the rows of this matrix. 00 if the complements of these vectors are added, 00,10,11,01 vectors are created. If the complements of these 4 vectors are taken and rearranged with their repeaters, 0011, 1001, 1100, 0110, 0000, 1010, 1111, 0101t he vectors are obtained. These vectors created by cluster one dual code indicates. This code length 4 is the number of elements 8 and 2 is the Hamming weight. Thus, M a Hadamard code is obtained with the Hadamard matrix. The found code is equal to $(n,k,d)=(4,8,2)$ the parameterized $\Phi({C_R}^{0,1})$ code.

Moreover $({C_R}^{0,1})^{\perp}=\{00,1+v1+v\}$ and $\Phi_1\left(({C_R}^{0,1})^{\perp}\right)=\{0000,1111\}$ happens. Apart from this ${C_R}^{0,1}$, $\tau_1({C_R}^{0,1})={C_R}^{0,1}$ the code is condition, because it provides. It is a cyclic code. Similar in the form the code $\sigma^{\otimes 2}(\Phi_1({C_R}^{0,1}))=\Phi({C_R}^{0,1})$ provides $\Phi({C_R}^{0,1})$. It is a first quasi-cyclic code order of two.

3.10 Definition : Let $\alpha_1,\alpha_2\geq 0$ and $n=2^{2\alpha_1+\alpha_2}$.

${C_R}^{\alpha_1,\alpha_2}=\{(c_1,c_2).N^{\alpha_1,\alpha_2}|\, c_1\in R^{\alpha_1+1}, c_2\in {F_2}^{\alpha_2}\}$a $(n,4n,n)$ parameter code on the ring R.

${S_R}'=\{00\ldots0,1+v1+v\ldots1+v\}$ a code is $(n,2,2n)$ parameter on the ring R. ${S_R}''=\{00\ldots0,11\ldots1,vv\ldots v,1+v1+v\ldots1+v\}$ is a code $(n,4,n)$ on the ring $R=F_2+vF_2$.

Let ${S_R}',{S_R}''$ and ${C_R}^{\alpha_1,\alpha_2}$ using codes on the ring R. ${}_1{C_R}^{\alpha_1,\alpha_2}=\{(a,a+b)|\, a\in {C_R}^{\alpha_1,\alpha_2},\ b\in {S_R}'\}$ is called 1st repetitive formation code

with parameter $(2n,8n,2n)$.

Again let ${S_R}',{S_R}''$and ${C_R}^{\alpha_1,\alpha_2}$ using on the ring R. ${}_2{C_R}^{\alpha_1,\alpha_2}=\{(a,a+b,a+v.b,a+(1+v).b)|\, a\in {C_R}^{\alpha_1,\alpha_2},\ b\in {S_R}''\}$ is called 2th repetitive formation code with parameter $(4n,16n,4n)$.

3.11 Proposition : ${C_R}^{\alpha_1,\alpha_2}$ is equivalent to ${}_2{C_R}^{\alpha_1+1,\alpha_2}$ the code , in case $\alpha_1,\alpha_2\geq 0$. When it happens specifically $\alpha_1=0$, it is ${}_2\,{C_R}^{\alpha_1,\alpha_2}\approx {C_R}^{\alpha_1+1,\alpha_2}$.

3.12 Example :

${C_R}^{0,1}=\{(c_1,c_2).{N_R}^{0,1}|\, c_1\in R,\ c_2\in F_2\}$

$=\{00,11,vv,1+v1+v,0v,11+v,v0,1+v1\}\subseteq R^2$

is code with $(2,8,2)$ parameter.

Let ${S_R}'=\{00,1+v1+v\}$.

${}_1{C_R}^{0,1}=\{(a,a+b)|\, a\in {C_R}^{0,1},\ b\in {S_R}'\}$

$$=\left\{\begin{array}{c}0000,001+v1+v,01+v01+v,\\ 01+v1+v0,1111,11vv,1v1v,\\ 1vv1,vvvv,vv11,v1v1,v11v,\\ 1+v1+v1+v1+v,\\ 1+v1+v00,1+v01+v0,1+v001+v\end{array}\right\}\subseteq R^4$$

is code with $(4,16,4)$ parameter. At the same time, this ${}_1{C_R}^{0,1}$ is equal to the code ${C_R}^{0,1}$. Similarly ${C_R}^{0,2}$ created obtained with the matrix ${N_R}^{0,2}$.

3.13 Example : ${C_R}^{0,1}=\{(c_1,c_2).{N_R}^{0,1}|\, c_1\in R,\ c_2\in F_2\}$

$=\{00,11,vv,1+v1+v,0v,11+v,v0,1+v1\}\subseteq R^2$ is code with $(2,8,2)$ parameter.

Let ${S_R}''=\{00,11,vv,1+v1+v\}$.

${}_2{C_R}^{0,1}=\{(a,a+b,a+v.b,a+(1+v).b)|\, a\in {C_R}^{0,1},\ b\in {S_R}''\}$

$$= \left\{ \begin{array}{c} 00000000{,}0011vv1+v1+v, \\ 00vv111+v1+v, \\ 001+v1+v1+v1+v00, \\ 01+v01+v01+v01+v, \\ 01+v1vv11+v0{,}01+vv11v1+v0, \\ 11111111{,}11001+v1+vvv, \\ 111+v1+v00vv, \\ 11vvvv11{,}1v1v1v1v, 1v01+v1+v0v1, \\ 1v1+v001+vv1{,}1vv1v11v, \\ vvvvvvvv, vv1+v1+v0011, \\ vv001+v1+v11, vv1111vv, \\ v1v1v1v1, \\ v11+v001+v1v, v101+v1+v01v, v11v1vv1, \\ 1+v1+v1+v1+v1+v1+v1+v1+v, \\ 1+v1+vvv1100{,}1+v1+v11vv00, \\ 1+v1+v00001+v1+v, \\ 1+v01+v01+v01+v0{,}1+v0v11v01+v, \\ 1+v01vv101+v, 1+v001+v01+v1+v0 \end{array} \right\} \subseteq R^8$$

is possible. The code is also ${N_R}^{1,1}$produced with the matrix $(8{,}32,8)$_parameterized ${C_R}^{1,1}$ to the code equal is found .

3.14 Example : ${C_R}^{1,0} = \{c_1.{N_R}^{1,0} | c_1 \in R^2\} =$

$$\left\{ \begin{array}{c} 0000{,}1111, vvvv, 1+v1+v1+v1+v, \\ 01v1+v, 101+vv, \\ v1+v01{,}1+vv10{,}0v0v, 11+v11+v, \\ v0v0{,}1+v11+v1, \\ 01+vv1{,}1v1+v0, v101+v, 1+v01v \end{array} \right\}$$

generated by generator matrix ${N_R}^{1,0} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & v & 1+v \end{bmatrix}$ that $(4{,}16,4)$ parameter is code. From here

$${}_1{C_R}^{1,0} = \{(a, a+b) |\ a \in {C_R}^{1,0},\ b \in {S_R}'\}$$

$$= \left\{ \begin{array}{c} 00000000{,}0011vv1+v1+v, 00vv111+v1+v, \\ 001+v1+v1+v1+v00, \\ 01+v01+v01+v01+v, \\ 01+v1vv11+v0{,}01+vv11v1+v0, \\ {,}11111111{,}11001+v1+vvv, \\ 111+v1+v00vv, \\ 11vvvv11{,}1v1v1v1v, \\ 1v01+v1+v0v1{,}1v1+v001+vv1{,}1vv1v11v, \\ vvvvvvvv, vv1+v1+v0011, \\ vv001+v1+v11, vv1111vv, \\ v1v1v1v1, \\ v11+v001+v1v, v101+v1+v01v, \\ v11v1vv1, \\ 1+v1+v1+v1+v1+v1+v1+v1+v, \\ 1+v1+vvv1100{,}1+v1+v11vv00, \\ 1+v1+v00001+v1+v, \\ 1+v01+v01+v01+v0{,}1+v0v11v01+v, \\ 1+v01vv101+v, 1+v001+v01+v1+v0 \end{array} \right\} \subseteq$$

R^8 is found . Like this suitable your ingredients place to change and permutation implementation with ${C_R}^{1,1}$code is obtained. So the first repetitive formation ${}_1{C_R}^{1,1}$ code of code ${C_R}^{1,0}$, also the second repetitive formation ${C_R}^{2,0}$code of ${C_R}^{1,0}$ to the code equivalent is seen .

IV. EXISTENCE OF ODD AND EVEN CODES

4.1 Definition : Let $\alpha_1, \alpha_2 \geq 0$. There be a code ${C_R}^{\alpha_1,\alpha_2} \subseteq R^n$ including $n = 2^{2\alpha_1+\alpha_2}$.

On the ring R, $even({C_R}^{\alpha_1,\alpha_2})$

$$= \left\{(c_0, c_2, \dots, c_{n-2}) \in R^{\frac{n}{2}} | (c_0, c_2, \dots, c_{n-1}) \in {C_R}^{\alpha_1,\alpha_2}\right\}$$

is defined to the code ${C_R}^{\alpha_1,\alpha_2}$ is called the even code. On the ring R, $odd({C_R}^{\alpha_1,\alpha_2})$

$$= \left\{(c_1, c_3, \dots, c_{n-1}) \in R^{\frac{n}{2}} | (c_0, c_2, \dots, c_{n-1}) \in {C_R}^{\alpha_1,\alpha_2}\right\}$$

form defined to the code ${C_R}^{\alpha_1,\alpha_2}$ is called the odd code.

Even and odd of codes definitions F_2 is given in a similar way on the object.

4.2 Proposition: Let ${C_R}^{\alpha_1,\alpha_2} \subseteq R^n$ a code.

i) for $\alpha_1 \geq 1, \alpha_2 \geq 0$,

$even({C_R}^{\alpha_1,\alpha_2}) = odd({C_R}^{\alpha_1,\alpha_2}) = {C_R}^{\alpha_1-1,\alpha_2+1}$

is possible.

ii) for $\alpha_1 \geq 0, \alpha_2 \geq 1$,

$even({C_R}^{\alpha_1,\alpha_2}) \approx odd({C_R}^{\alpha_1,\alpha_2}) = {C_R}^{\alpha_1,\alpha_2-1}$

is possible.

4.3 Proposition : Let be $C^{\alpha_1,\alpha_2} \subseteq R^n$a code and Φ be the Gray transformation on R^n. In this case, $even\left(\Phi({C_R}^{\alpha_1,\alpha_2})\right) = \Phi\left(even({C_R}^{\alpha_1,\alpha_2})\right)$ is provided.

Also 4.3 proposition, It is provided for odd codes .

V. APPLICATIONS OF CODES

5. 1 Example : the ${}_1{C_R}^{0,1}$code to equal ${C_R}^{0,2}$ code has been found like following .

$${C_R}^{0,2} = \left\{\begin{matrix} 0000,01+v01+v,001+v1+v, \\ 01+v1+v0,1vv1, \\ 11vv,1v1v,1111,vvvv,v1v1, \\ vv11,v11v,1+v001+v, \\ 1+v1+v00,1+v01+v0, \\ 1+v1+v1+v1+v \end{matrix}\right\} \subseteq R^4$$

of this even code, $even({C_R}^{0,2})$

$= \{00,01+v,1v,11,vv,v1,1+v0,1+v1+v\}$

is possible . From here Gray image of the code

$\Phi\left(even({C_R}^{0,2})\right) =$

$\{0000,0101,1001,1100,0011,0110,1010,1111\}$

is found as . This code is the odd code

$odd({C_R}^{0,2}) = \{00,1+v1+v,01+v,1+v0,1v,v1,vv,11\}$ is possible and This Gray image of the code

$\Phi\left(odd({C_R}^{0,2})\right) =$

$\{0000,1111,0110,0101,1001,1100,0011,1010\}$

aspect is found .

${C_R}^{0,2}$ code for even code and odd code is equal .

Moreover ${C_R}^{0,2}$ Gray image of the code

$$\Phi({C_R}^{0,2}) = \left\{\begin{matrix} 00000000,01010101,00110011,01100110, \\ 10010110,11000011,10100101,11110000, \\ 00001111,01011010,00111100,01101001, \\ 10011001,11001100,10101010,11111111 \end{matrix}\right\} \subseteq {F_2}^8$$

is possible . Even code of this code

$even\left(\Phi({C_R}^{0,2})\right) =$

$\{0000,0101,0011,0110,1111,1010,1100,1001\}$It is possible. Then $even\left(\Phi({C_R}^{0,2})\right) =$ $\Phi\left(even({C_R}^{0,2})\right)$equality is obtained.

5.2 Example : For ${C_R}^{1,0}$ code, even code and odd code equal . Moreover this codes ${C_R}^{0,1}$ to the code equal ${C_R}^{1,0} = \{c_1.{N_R}^{1,0} | c_1 \in R^2\}$

$$= \left\{\begin{matrix} 0000,1111,vvvv,1+v1+v1+v1+v, \\ 01v1+v,101+vv, \\ v1+v01,1+vv10,0v0v, \\ 11+v11+v,v0v0,1+v11+v1, \\ 01+vv1,1v1+v0,v101+v,1+v01v \end{matrix}\right\}$$

is possible. From here ,

$$even({C_R}^{1,0}) = \{00,11,vv,1+v1+v,0v,11+v,v0,1+v1\}$$

$$odd({C_R}^{1,0}) = \{00,11,vv,1+v1+v,0v,11+v,v0,1+v1\}$$

Moreover ${C_R}^{0,1} = \{00,11,vv,1+v1+v,0v,11+v,v0,1+v1\}$ of the code are the elements. In that case $even\left({C_R}^{1,0}\right) = odd\left({C_R}^{1,0}\right) = {C_R}^{0,1}$ is shaped like .

5.3 Example : The ${}_2{C_R}^{1,0}$ code is equal to ${C_R}^{1,0} = \left\{c_1.{N_R}^{1,0} | c_1 \in R^2\right\}$ in case $n = 2^{2\alpha_1 + \alpha_2}$.

${C_R}^{1,1} = \{(c_1, c_2).N^{1,1} | \, c_1 \in R^2, c_2 \in F_2\}$ is code with (8,32,8) parameter. From here, even code of ${C_R}^{1,1}$code $even\left({C_R}^{1,1}\right)$

$$= \left\{ \begin{matrix} 0000, 0v0v, 00vv, 0vv0, 1111, \\ 11 + v11 + v, 111 + v1 + v, \\ 11 + v1 + v1, 1 + v1 + v1 + v1 + v, \\ 1 + v11 + v1, \\ 1 + v1 + v11, 1 + v111 + v, vvvv, v0v0, \\ v101 + v, v1 + v01 \end{matrix} \right\} \subseteq R^4$$

aspect is found. This code has

(4,16,4) parameter . This code suitable place changing and permutation when applied

$${}_2{C_R}^{1,0} = \left\{ \begin{matrix} 0000, 1111, vvvv, 1 + v1 + v1 + v1 + v, \\ 01v1 + v, 101 + vv, \\ v1 + v01, 1 + vv10, 0v0v, 11 + v11 + v, \\ v0v0, 1 + v11 + v1, \\ 01 + vv1, 1v1 + v0, v101 + v, 1 + v01v \end{matrix} \right\}$$

get the code is done.

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