Uluslararası İleri Doğa Bilimleri ve Mühendislik Araştırmaları Dergisi
Sayı 7, S. 278-282, 11, 2023

Araştırma Makalesi

IJANSER

https://alls-academy.com/index.php/ijanser
ISSN: 2980-0811

International Journal of Advanced Natural Sciences and Engineering Researches
Volume 7, pp. 278-282, 11, 2023

Research Article

Control on the Sensor Coherence of Transmon Superconducting Qubit via the Gradient Descent Algorithm

Sergey Borisenok[*1,2]

[1]*Department of Electrical and Electronics Engineering, Faculty of Engineering, Abdullah Gül University, Kayseri, Türkiye*
[2]*Feza Gürsey Center for Physics and Mathematics, Boğaziçi University, İstanbul, Türkiye*

[*]*(sergey.borisenok@agu.edu.tr) Email of the corresponding author*

(Received: 19 December 2023, Accepted: 25 December 2023)

(3rd International Conference on Scientific and Academic Research ICSAR 2023, December 25-26, 2023)

ATIF/REFERENCE: Borisenok, S. (2023). Control on the Sensor Coherence of Transmon Superconducting Qubit via the Gradient Descent Algorithm. *International Journal of Advanced Natural Sciences and Engineering Researches*, 7(11), 278-282.

Abstract **–** Quantum systems based on superconducting circuits with qubits with quantized energy levels serve as an efficient physical background for constructing different quantum devices, for instance, quantum sensors. The important class of such engineering devices belongs to the systems with quantum tunneling effects, like Josephson junctions, where the set of energy levels is influenced by the external fields. These fields change the phase properties of the measuring qubits, and, in this way, they can be detected. The weakly anharmonic oscillator is often referred as a transmon qubit. Such qubits are constructed with superconducting capacitor structures connected by Josephson junctions, they can be controlled by microwave pulses of a few nanoseconds. The quantum sensing mechanism studied here is based on coupling the measuring transmon superconducting qubits with magnetic materials. The sensing protocol of Ramsey Fringes interferometry covers several stages and demands high-quality control over the sensor coherence. Although the weakly nonlinear properties of transmons prevent the sensing procedure from the strong influence of external perturbations, nevertheless, the extension of the coherence time interval and minimization of the dephasing effects are still a matter of great importance. Here we discuss the improvement of the quantum sensor performance by the application of feedback control in the form of gradient descent algorithm over the dephasing factor-function.

Keywords – Quantum Sensors, Transmon Superconducting Qubit, Sensor Coherence, Dephasing, Gradient Descent Algorithm.

I. INTRODUCTION

Quantum systems based on superconducting circuits with qubits with quantized energy levels serve as an efficient physical background for constructing different quantum devices, for instance, quantum sensors [1]. The important class of such engineering devices belongs to the systems with quantum tunneling effects, like Josephson junctions, where the set of energy levels is influenced by the external fields. These fields change the phase properties of the measuring qubits, and, in this way, they can be detected.

1.1. Transmon Superconducting Qubits

Level-2 and level-3 headings can be used to detail main headings.

Superconducting qubits have many advantages over elements which can be described as quantum harmonic oscillators (QHOs), the standard objects to study in quantum mechanics. The comparison is made in Fig.1 for the quantum harmonic oscillator based on a parallel LC-circuit and transmon superconducting qubit based on a Josephson junction.

In Fig. 1 one can see that the energy levels for the QHO (a) the energy levels are spaced equidistantly shaped by the quadratic potential (b) forbidding us from addressing the transitions individually, while for the superconducting qubit (c) the potential is sinusoidal (d). This property serves for the isolation of the two lowest energy levels $|0\rangle$ and $|1\rangle$ for the purpose of computation or sensing. It also makes the qubit to be less sensitive to the external noise.

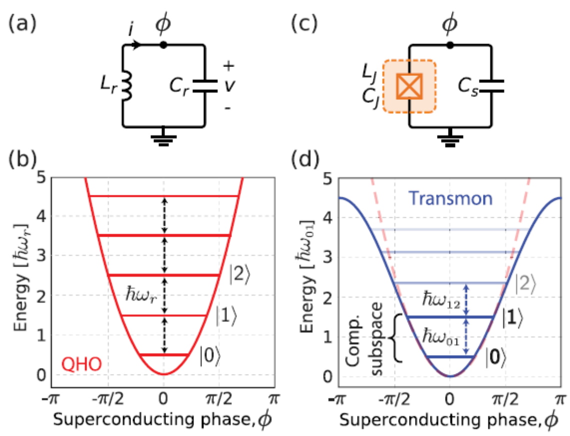

Fig. 1. Comparison of a parallel LC-circuit (a) and its energy potential vs the phase (b) with a superconducting qubit (c) and its energy potential vs the phase (d) [2].

Fig.1, in fact, is the comparison of a linear quantum circuit element (a,b) with a nonlinear one (c,d). The weakly anharmonic oscillator is often referred as a *transmon* qubit. Such qubits are constructed with superconducting capacitor structures connected by Josephson junctions [3], they can be controlled by microwave pulses of a few nanoseconds [4].

1.1.Superconducting Qubit-Based Quantum Sensors

Due to their nonlinear properties, transmon superconducting qubits can be used to sense weak external magnetic fields [5].

As an example, let's take the ***Ramsey Fringes interferometry*** scheme, which includes a few algorithmic phases:

1. *First control pulse* for the preparation of sensing: The sensing qubit is prepared as a superposition of its pure quantum states $|0\rangle$ and $|1\rangle$;
2. *Interaction with the external field*: At this stage, the sensing qubit accumulates the phase change due to the external field interaction;
3. *Second control pulse* which is identical to the first one, but *with some time delay*: Preparing for the measuring procedure;
4. Measuring *the population of qubit basic states*: This phase restores the external field via the accumulated phase.

The principal scheme for Ramsey fringes interferometry of external magnetic field is presented in Fig.2 (let us call it the Danilin-Nugent-Weides, or DNW protocol) [6].

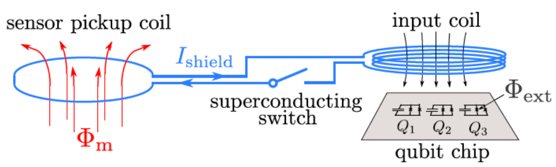

Fig. 2. Basic setup for superconducting multi-qubit-based sensor [6].

This scheme consists of the following principal parts:

- *Sensor pickup coil* serves as an input circuit where the external magnetic flux Φ_m is sensed and converted gradually into the magnetic flux Φ_{ext};
- *Superconducting switch* allows for the application of the external magnetic flux only

during the target period, with fast on/off switching characteristics;

- *Input coil* for the appearance of the shielding current I_{shield};
- *Qubit chip*, performing the phase manipulation for the sensing protocol.

The current I_{shield} calls the external magnetic flux Φ_m with the discrete set of the quantized fluxes $n\Phi_0$, with the integer n.

To provide the optimal flux sensing regime with different qubit transition frequencies, the DNW protocol is based on tunable superconducting transmon qubits.

The quantum sensing mechanism studied here follows our feedback control approach developed in [7-9], and it is based on coupling the measuring qubits with magnetic materials.

II. Control over the Sensor Coherence

As one can see from the description of the sensing protocol, the Ransey Fringers interferometry deals with the calibration patterns before the field measuring phase. For the second control pulse, the longer if the delay time, the more the protocol is sensitive to the external flux. ***The delay time must be shorter than the coherence time for the sensing qubits*** [6]. For that reason, it is extremely important to extend the coherence time to improve the performance of the sensor, I.e. to be able to control the effects of decoherence. The important part of such control is minimization of the effects coming from dephasing.

2.1. Dephasing

The coherence of the DNW sensor depends on the relaxation and the rate of dephasing for a transmon qubit. If f stays for the 0-1 transition frequency in the qubit, and the external magnetic flux is Φ, then (based on [6]):

$$f = \left(\frac{E_C + E_J}{h}\right)\sqrt{\left|\cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right|} - \frac{E_C}{h} \quad . \qquad (1)$$

Here Φ_0 is the magnetic flux quantum. The frequency f contains two energy parameters: the qubit charging energy E_C and the maximal Josephson energy E_J. To avoid the influence of environmental noise and other fluctuations (including fabrication defects), the transmon qubits must be designed in the regime $E_J > E_C$ [4].

The factors that influence the dephasing process are:

- the quasiparticle tunneling,
- the charge noise,
- the flux noise,
- the critical current noise.

The first two channels of dephasing are not critical for transmon-based devices due to the exponentially fast decay in the charge dispersion [10].

The flux noise decay versus time can be described via the factor-function [11]:

$$D_F = \exp\left\{-\frac{\pi^4\alpha^2 t^2\cdot\left(\frac{E_C+E_J}{h}\right)^2 \sin^2\left(\pi\frac{\Phi}{\Phi_0}\right)}{\cos\left(\pi\frac{\Phi}{\Phi_0}\right)}\right\}\times$$

$$\times\exp\left\{-\frac{\pi^5\alpha^2 t\cdot\left(\frac{E_C+E_J}{h}\right)\left[1+\cos^2\left(\pi\frac{\Phi}{\Phi_0}\right)\right]}{2\cos^{3/2}\left(\pi\frac{\Phi}{\Phi_0}\right)}\right\} . \qquad (2)$$

Here α is a phenomenological parameter related to the flux noise power spectral density $S_\Phi(f)$ [12]:

$$\alpha = \frac{A_\Phi}{\Phi_0}\ , \text{ where } S_\Phi(f) = \frac{A_\Phi^2}{f}\ . \qquad (3)$$

The critical current noise due to the fluctuation to the maximum Josephson energy E_J leads to the factor-function [6]:

$$D_C = \exp\left\{-\pi^2\gamma^2 t^2\cdot\left(\frac{E_C+E_J}{h}\right)^2 \cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right\} . \qquad (4)$$

The constant γ is defined from the Josephson current I_c noise power spectral density as [12]:

$$\gamma = \frac{A_C}{I_c}\ , \text{ where } S_C(f) = \frac{A_C^2}{f}\ . \qquad (5)$$

Thus, the total factor-function for the dephasing process can be presented as a product of (2) and (4):

$$D = D_F D_C . \qquad (6)$$

Our task is to minimize the decay (6), i.e. to minimize the arguments of the exponents.

2.2.Minimization of the Dephasing Factor-Function

Minimization of the decay in (6) corresponds to the minimization of the sum of all arguments under exponents. Because the control parameters E_C and E_J stand in (2) and (4) in the same combination multiplied by the time t, let's take the tracking control signal in the dimensionless form:

$$u(t) = \frac{t \cdot (E_C + E_J)}{h} . \qquad (7)$$

The goal function for the minimization is:

$$G_u = A_u u^2 + B_u u + C_u , \qquad (8)$$

where

$$A_u = \frac{\pi^4 \alpha^2 \sin^2\left(\pi \frac{\Phi}{\Phi_0}\right)}{\cos\left(\pi \frac{\Phi}{\Phi_0}\right)} + \pi^2 \gamma^2 \cos\left(\pi \frac{\Phi}{\Phi_0}\right) ;$$

$$B_u = \frac{\pi^5 \alpha^2 \left[1 + \cos^2\left(\pi \frac{\Phi}{\Phi_0}\right)\right]}{2\cos^{3/2}\left(\pi \frac{\Phi}{\Phi_0}\right)} , \qquad (9)$$

and C_u is a positive normalization constant: the polynomial (8) for the decay rate cannot be negative. By (7)-(8), $D = \exp\{-G_u\}$.

Usually in the experiments, the variations of the external flux cover the first half of the period: $\Phi/\Phi_0 < 0.5$, such that sin and cos in (9) are not negative. The control u in (7) is non-negative as well. Thus, indeed, the goal function (8) is defined as non-negative.

The position of the absolute minimum $-B_u/2A_u$ is negative and cannot be achieved in the control process. Apart from this, the energy E_J may fluctuate in time. For that reason, we apply here gradient descent algorithm (GDA) to find the actual minimum of the goal function G_u.

2.3.Gradient Descent Algorithm

The function (8) is differentiable and convex for u, so we can apply GDA in its 1-dimensional form. Let's define the process as [13]:

$$u_{k+1} = u_k - \eta \left.\frac{\partial G_u(u)}{\partial u}\right|_{u=u_k} , \qquad (10)$$

with a positive constant η (learning rate). Eq.(8) implies:

$$u_{k+1} = u_k - \eta \cdot (2A_u u + B_u) . \qquad (11)$$

Replacing (11) with the differential formulation:

$$\frac{du}{dt} = -\eta \cdot (2A_u u + B_u) . \qquad (12)$$

we obtain the solution:

$$u(t) = u_0 e^{-\eta t} + \frac{B_u}{2A_u}\left(e^{-\eta t} - 1\right) ; \text{ where } u_0 = u(0) . \qquad (13)$$

According to our physical model, (13) cannot be negative. That implies the inequality for the time interval:

$$t \le t_{\max} = \frac{1}{\eta} \ln\left(1 + \frac{2A_u u_0}{B_u}\right). \qquad (14)$$

For small u_0 we can evaluate (14) as $t_{\max} \cong 2A_u u_0 / \eta B_u$. Thus, GDA for the sensing process is valid for the time range [0, $t_{\max}$], which depends on the external magnetic flux Φ (via the parameters A_u and B_u).

The typical experimental values for α and γ are $\alpha = \gamma = 10^{-6}$ [6]. Then by (9) we can plot the parameter $K = (2A_u/B_u)\cdot 10^{12}$ versus Φ/Φ_0:

$$K = \frac{4 \cdot 10^{12}}{\pi^3} \cdot \sqrt{\cos\left(\pi \frac{\Phi}{\Phi_0}\right)} \cdot \frac{1 + (\pi^2 - 1) \cdot \sin^2\left(\pi \frac{\Phi}{\Phi_0}\right)}{1 + \cos^2\left(\pi \frac{\Phi}{\Phi_0}\right)} . \qquad (15)$$

Eq.(15) is plotted in Fig.3 for the range $\Phi/\Phi_0 < 0.5$.

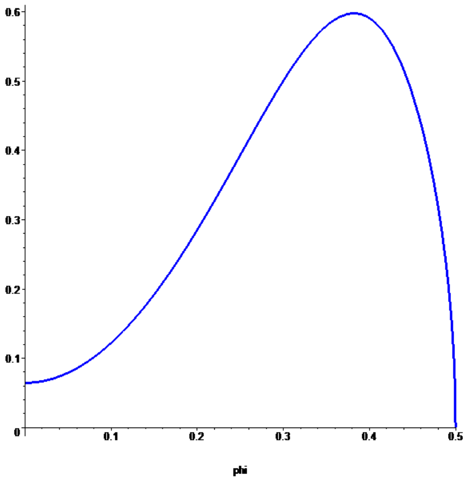

Fig. 3. The parameter K = $(2A_u/B_u)\cdot 10^{12}$ versus Φ/Φ_0.

Thus, the upper limit for t_{max} is about $6\cdot 10^{-13} u_0/\eta$.

III. Results

The minimization of dephasing effects by the application of the gradient descent algorithm improves the coherence properties of the sensing transmon superconducting qubit.

IV. DISCUSSION

The gradient descent developed here and other gradient algorithms [14] can be redesigned for different types of quantum sensors. Alternative approaches for feedback control (target attractor 'synergetic' control [15] and others) will be the subject of our further research.

V. CONCLUSION

The application of feedback control methods can improve sufficiently the performance of magnetic field sensors based on transmon superconducted qubits, and it is extremely efficient for minimization of dephasing effects in Ramsey fringes interferometry protocols.

Acknowledgment

Our research has been supported by the Research Fund of Abdullah Gül University; Project Number: BAP FBA-2023-176 "Improving efficiency of qubit-based sensors via feedback control algorithms".

References

[1] C. L. Degen, F. Reinhard, P. Cappellaro, "Quantum sensing", *Review of Modern Physics*, vol. 89, p. 035002, 2017.

[2] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, W. D. Oliver, "A quantum engineer's guide to superconducting qubits", *Applied Physics Reviews*, vol. 6, p. 021318, 2019.

[3] Y. Y. Gao, M. A. Rol, S. Touzard, C. Wang, "Practical guide for building superconducting quantum devices", *PRX Quantum*, vol. 2, p. 040202, 2021.

[4] M. Kjaergaard, M. E. Schwartz, J. Braumüller, P. Krantz, J. Wang, S. Gustavsson, W. D. Oliver, "Superconducting qubits: Current state of play", arXiv:1905.13641v3 [quant-ph], 2020.

[5] A. Kringhøj, T. W. Larsen, O. Erlandsson, W. Uilhoorn, J. G. Kroll, M. Hesselberg, R. P. G. McNeil, P. Krogstrup, L. Casparis, C. M. Marcus, K. D. Petersson, "Magnetic-field-compatible superconducting transmon qubit", *Physical Review Applied*, vol. 15, p. 054001, 2021.

[6] S. Danilin, N. Nugent, M. Weides, "Quantum sensing with tunable superconducting qubits: optimization and speed-up", arXiv:2211.08344v3 [quant-ph], 2022.

[7] S. Borisenok, "Control over performance of qubit-based sensors", *Cybernetics and Physics*, vol. 7(3), p.p. 93-95, 2018.

[8] S. Borisenok, "Feedback control over quantum sensing based on Bose-Einstein condensate trapped in two-dimensional ring potential", *International Journal of Advanced Natural Sciences and Engineering Researches*, vol. 7(10), p.p. 207-211, 2023.

[9] S. Borisenok, "Sensing magnetic field with single-spin dynamical probe state: Control over sensing precision via quantum Fisher information", *European Journal of Science and Technology*, vol. 48, p.p. 29-33, 2023.

[10] J. Koch, T. M. Yu, J. Gambetta,, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, R. J. Schoelkopf, "Charge-insensitive qubit design derived from the Cooper pair box", *Physical Review A*, vol. 76, p. 042319, 2007.

[11] D. J. Van Harlingen, T. L. Robertson, B. L. T. Plourde, P. A. Reichardt, T. A. Crane, J. Clarke, "Decoherence in Josephson-junction qubits due to critical-current fluctuations", *Physical Review B*, vol. 70, p. 064517, 2004.

[12] F. Yoshihara, K. Harrabi, A. O. Niskanen, Y. Nakamura, J. S. Tsai, "Decoherence of Flux Qubits due to 1/f Flux Noise", *Physical Review Letters*, vol. 97, p. 167001, 2006.

[13] S. Boyd, L. Vandenberghe, *Convex Optimization*, Cambridge: Cambridge University Press, 2004.

[14] A. L. Fradkov, *Cybernetical Physics. From Control of Chaos to Quantum Control*. Berlin: Springer, 2017.

[15] A. A. Kolesnikov, "Introduction of synergetic control" in *Proc. 2014 American Control Conference*, Portland, USA, p.p. 3013-3016, 2014.