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Bezier Curve Fitting of Blade Profile considering Curvature Variation

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Abstract – This paper presents a single C^2-continuous Bezier curve fitting procedure for a blade profile. The fitting procedure is based on nonlinear least squares approach and it also contains fairing term to achieve a minimum variation in the curvature of fitted Bezier curve. The Levenberg-Marquardt method is employed to find Bezier curve control points and parameters for each data points. For validation, a well-known LS89 blade profile data is used and a 12th degree single Bezier curve is obtained via proposed procedure for this blade profile. Thanks to the fitting procedure, we are able to represent LS89 blade profile, accommodating 405 data points, with only 13 control points, which is a quite efficient way in terms of blade profile design and optimization processes, and determination of manufacturing tolerances. Thus, the proposed fitting procedure has a great potential for practical applications.

Keywords – Aero-Engine, Blade, Inspection, Reverse Engineering, Geometry Fitting

I. Introduction

In aero-engines, blades are one of the most crucial components [1, 2]. Their free-form complex shapes directly determine efficiency, performance and safety of the aero-engines. Thus, their two-dimensional profiles are undergone an optimization during the design process and an inspection after the manufacturing. All those require to obtain blade profile from measured data points. Ghorbani and Khameneifar [3], for instance, extracted blade profile from a noisy data acquired by 3D scanning. A strategy developed by Yun et al. [4] was introduced for reconstructing of blade profile from measured data points. Another study on reconstructing blade profile was conducted by Khameneifar and Feng [5]. In [6], a blade measurement procedure was developed to extract blade design parameters and geometric tolerances from measured data. Many attempts on this topic can be also found in Refs. [7-9]. All those studies underline the importance of the blade profile reconstruction or representation. Thus, in this paper, a single C^2-continuous Bezier curve fitting procedure considering curvature variation is proposed through nonlinear least squares approach. From now on, the paper is organized as follows; Section II contains a brief description of Bezier curve fitting. Section III shows the fitting results. Finally, Section IV concludes paper.

II. Bezier Curve Fitting

A general description of a Bezier curve [10] is given in Eq. (1).

$$Q(u) = \sum_{i=0}^{m} B_{i,m}(u)\, CP_i \qquad (1)$$

In this equation, m is the degree, CP_i is the ith control point, $u \in [0,1]$ is the parameter and $B_{i,m}(u)$ is the Bernstein polynomials computed as follows.

$$B_{i,m}(u) = \frac{m!}{i!\,(m-i)!}u^i(1-u)^{m-i} \quad (2)$$

For achieving a single continuous Bezier curve representation of a blade profile, C^2-continuity is satisfied. This also reduces the number of control points to be determined through nonlinear least squares fitting procedure.

In the nonlinear least squares fitting procedure, the parameter, p , dependent objective function given below is minimized [11].

$$\epsilon(p) = \frac{1}{2}\left(\sum_{j=1}^{N}\left[x_j^{data} - x_j^{BC}(p)\right]^2 + \sum_{j=1}^{N}\left[y_j^{data} - y_j^{BC}(p)\right]^2\right) + \beta * F_m \quad (3)$$

In Eq. (3), x_j^{data}, y_j^{data} and $x_j^{BC}(p)$, $y_j^{BC}(p)$ are the x and y-coordinates of blade profile data and Bezier curve fit, respectively, and N is the number of data points, β is a constant. F_m is the fairing function [12] given by

$$F_m = \int_0^L \left\|\frac{d\kappa}{ds}\right\|^2 ds \quad (4)$$

where κ is the curvature of the Bezier curve and it is computed as follows.

$$\kappa(u) = \frac{Q'(u) \times Q''(u)}{\|Q'(u)\|^3} \quad (5)$$

The objective function seen in Eq. (3) is iteratively minimized using Levenberg-Marquardt method [13].

III. Results

For validation, LS89 blade profile data [14], which is shown in Fig. 1, is used. The data contains totally 405 points. By employing the proposed procedure, we are able to obtain a single C^2-continuous 12th Bezier curve fit for this data, as seen in Fig. 2. From this figure, it is clearly observed that the fitted Bezier curve successfully represents the blade profile data. Specifically, 405 data points are represented only 13 control points. This shows the efficiency of the proposed fitting procedure. It is noteworthy that the constant for fairing function is $\beta = 8 \times 10^{-4}$ in this fitting.

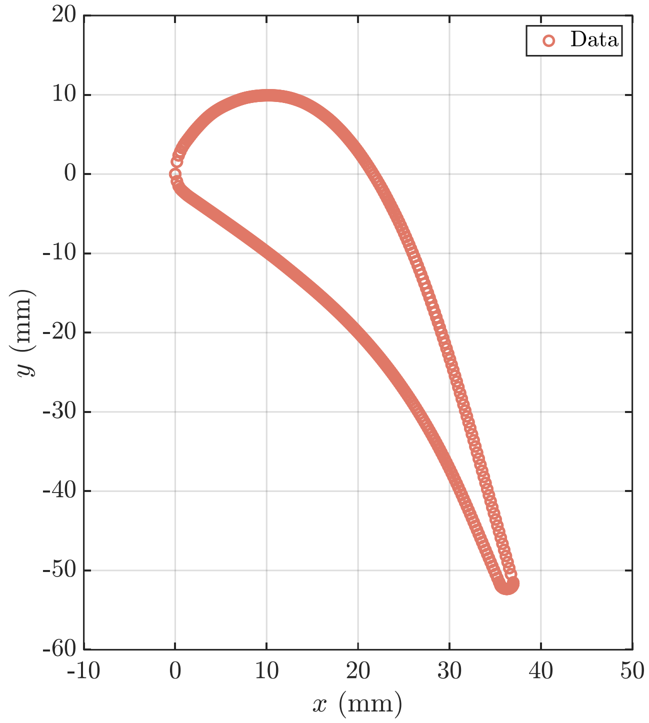

Fig. 1 LS89 blade profile data

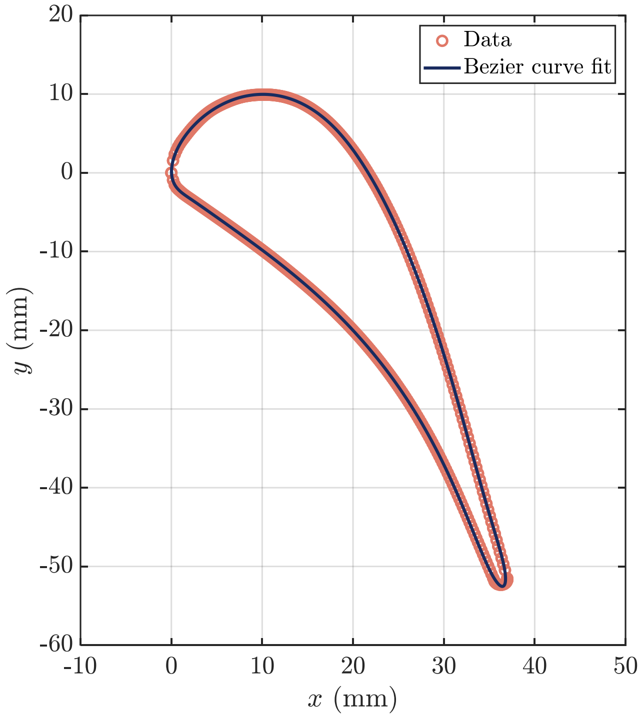

Fig. 2 Bezier fit of LS89 blade profile

To ensure that the fitted Bezier curve does not have any wiggles or non-smooth portions, the curvature combs are computed and they are plotted in Fig. 3. Based on these curvature combs, it can be stated that the fitted Bezier curve is quite smooth.

This could be easily verified by a closer look at the leading and trailing edges.

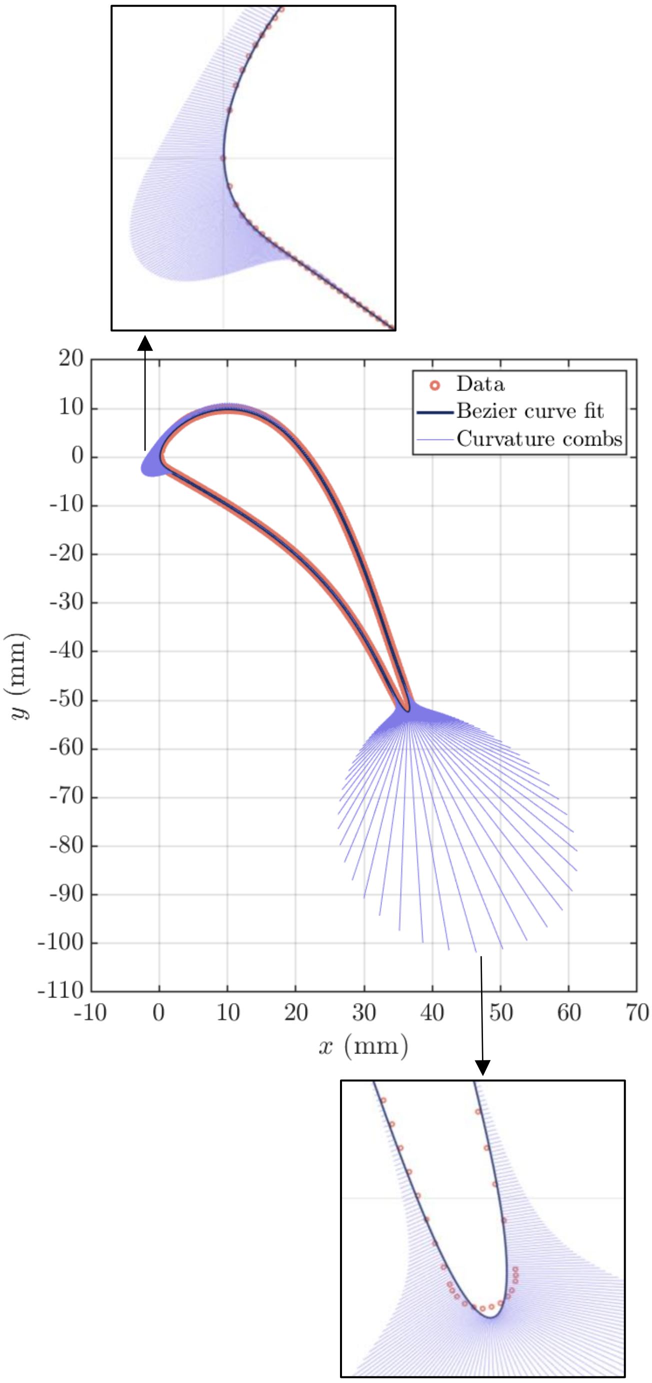

Fig. 3 Curvature combs of fitted Bezier curve with fairing

For better understanding of effect of the fairing process included in the proposed fitting procedure, the same fitting is also completed with the constant $\beta = 0$. It means that the fairing term is ignored. By doing so, it is obtained a Bezier fit shown in Fig. 4. As can be seen, there is a wiggle in the trailing edge, which is not acceptable. Thus, for a smooth fitting, a fairing process is essential.

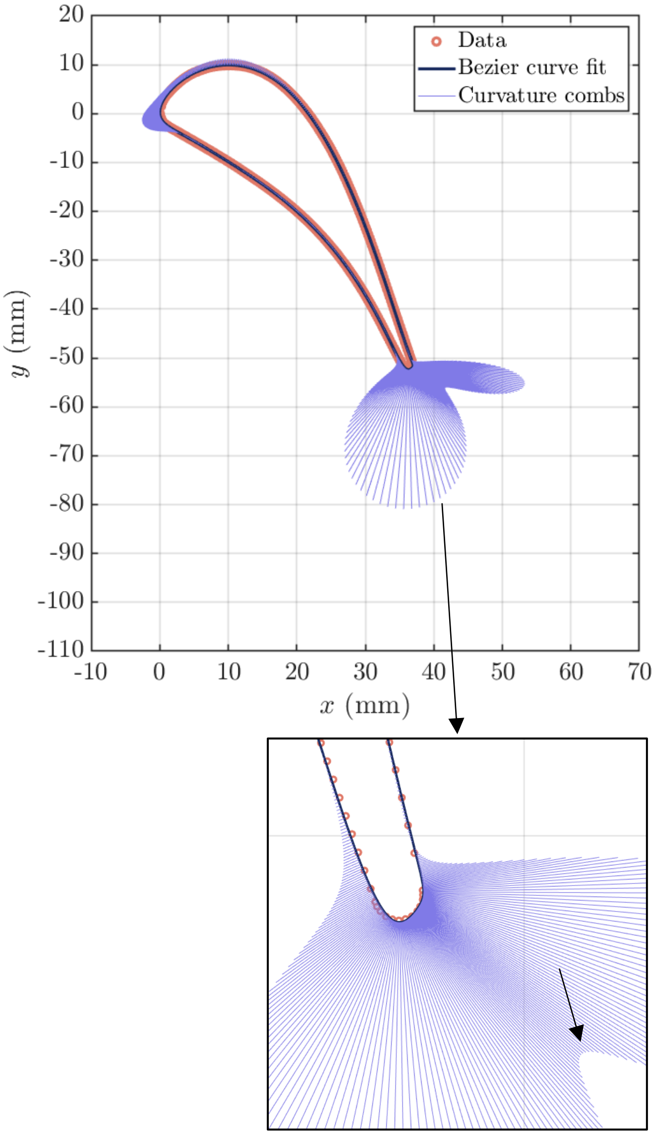

Fig. 4 Curvature combs of fitted Bezier curve without fairing

IV. CONCLUSIONS

This paper provided an efficient Bezier curve fitting procedure for a blade profile. The nonlinear least squares approach along with the Levenberg-Marquardt method was employed to find a Bezier curve fit having minimum curvature variation. Based on results and experience, a single Bezier curve fitting with minimum curvature variation is a quite useful approach for representing or reconstructing blade profile from measured data. The fitted Bezier curve might be used in blade profile design and optimization processes, machining tool path generation and inspection after manufacturing.

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